Linear expansion: $\ell = \ell_0(1+\alpha\Delta T)$

$$\Delta \ell = \alpha \ell_0 \Delta T$$

Volume expansion: $V(T) = \ell_0 w_0 h_0 (1+\alpha\Delta T)^3$

$$\Delta V = 3\alpha V_0 \Delta T$$
$$= \beta V_0 \Delta T$$

★ $\beta = 3\alpha$

$\gamma = 2\alpha$

IDEAL GAS LAW: $\boxed{PV = nRT}$

moles (n), 8.314 J/mol·K (R)

$PV = NkT$ (k: 1.38e-23)

$$nR = \frac{PV}{T} = \frac{P'V'}{T'}$$

$$\Delta P = P' - P = \rho\, h$$

Force & Energy

$\Delta(mv) = 2mv_x$

$t = \frac{2l}{v_x}$

$F = \frac{\Delta(mv)}{\Delta t} = \frac{\Delta p}{\Delta t}$

$F = \frac{m}{\ell} N \overline{v_x^2}$ net force wall

$P = \frac{F}{A}$

avg speed

$PV = \frac{2}{3} N\left(\frac{1}{2} m\overline{v^2}\right)$

$\overline{v}$ = avg of speeds

$\overline{K} = \frac{1}{2} m\overline{v^2} = \frac{3}{2} kT$

$\overline{v} = \langle v \rangle$
$= \int v f(v)\, dv$

$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}$

$\int_0^\infty f(v)\, dv = N$

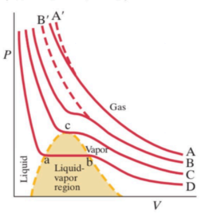


FIGURE 18–4 *PV* diagram for a real substance. Curves A, B, C, and D represent the same substance at different temperatures ($T_A > T_B > T_C > T_D$).

curve A: behavior of real gas
curve B: real gas at lower temps
curve C: behavior @ critical temperature at same temp
curve D: when liquefication occurs

Phase diagram to compare diff phases

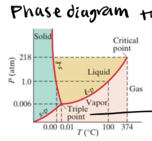


FIGURE 18–5 Phase diagram for water (note that the scales are not linear).

sublimation: solid to vapor

triple point: 3 phases exist tgt in equilibrium

superfluidity: zero viscosity & has strange properties

Van Der Waal

$$P = \frac{RT}{\left(\frac{V}{n}\right) - b} - \frac{a}{\left(\frac{V}{n}\right)^2}$$

$$\left(P + \frac{a}{\left(\frac{V}{n}\right)^2}\right)\left(\frac{V}{n} - b\right) = RT$$

Mean free path: $\ell_m = \frac{1}{4\pi r^2 \left(\frac{N}{V}\right)}$

← # of molecules

$4\pi r^2$: cross sectional area

Energy (E_{int})

Monoatomic: $E_{int} = N\left(\frac{1}{2} m\overline{v^2}\right)$
$= \frac{3}{2} nRT = \frac{3}{2} NkT$

Diatomic: $E_{int} = \frac{5}{2} nRT = \frac{5}{2} NkT$

$E_{int} = Q - W$ $\qquad dE_{int} = dQ - dW$

Heat (Q)

★ heat lost = heat gained

$Q = mc\Delta T = nC_v\Delta T$ (volume)
$= nC_p\Delta V$ (pressure)

$C_v = \frac{3}{2}R$ monoatomic

$C_v = \frac{5}{2}R$ diatomic

$C_p = C_v + R$

$Q = mL$ ← change of phase

★ remember to find heat change before AND after phase change

Types of processes:

Isothermal: $\Delta T = 0 \rightarrow \Delta E_{int} = 0 \rightarrow W = Q$

Adiabatic: $Q = 0$ (no heat flow in/out) $\qquad \Delta E_{int} = Q - W = 0 - W$
$= -W$

Isobaric = constant pressure
Isovolumetric = constant volume

$$\boxed{W = \int_{V_A}^{V_B} P\,dV} = P\Delta V$$

Isothermal: $W = \int_{V_A}^{V_B} P\,dV = nRT\int_{V_A}^{V_B}\frac{dV}{V} = nRT\ln\frac{V_B}{V_A}$

Isovolumetric: $W = 0$ $\quad (dV = 0)$

Isobaric: $W = \int_{V_A}^{V_B} P\,dV = P_B(V_B - V_A) = P\Delta V$

★ $\Delta E_{int} = 0$ in free expansion

Adiabatic : $Q=0$

$PV^{\gamma} = \alpha$ ← constant

$\gamma = \frac{C_p}{C_v} = 1.4$ diatomic
1.66 monoatomic

$dE_{int} = dQ - dW = -dW = -PdV$
$= nC_v dT$

$P_b V_b^{\gamma} = P_c V_c^{\gamma}$

Conduction

$\frac{Q}{t} = kA\frac{T_1 - T_2}{l} \rightarrow \frac{dQ}{dt} = -kA\frac{dT}{dx}$

Engines

Efficiency $e = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$

Heat input: $Q_H = W + Q_L$

Carnot engine: inverting

$e_{ideal} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$ ← kelvin

$COP = \frac{Q_L}{W}$ ← goal = cold

$= \frac{Q_L}{Q_H - Q_L}$

$COP_{ideal} = \frac{T_L}{T_H - T_L}$

$W = \frac{Q}{COP}$

Entropy

$\Delta S = \int \frac{dQ}{T} = \frac{Q}{T}$

$\Delta S = 0$ reversible
$\Delta S > 0$ irreversible

$\Delta S = \Delta S_H + \Delta S_L = -\frac{Q}{T_{HM}} + \frac{Q}{T_{LM}}$

↑ intermediate

$\Delta S = \frac{\Delta Q}{T_{LM}}$

Radiation

$\sigma = 5.67 \times 10^{-8}$

$\frac{dQ}{dT} = P_{out} = \varepsilon \sigma A T^4$

$\varepsilon = 1$

↑ surface area

Solar constant $S = \frac{P}{S}$ ← surface area

Rate of radiation: $P = S\pi r^2$

↑ solar constant ↑ area

$\frac{dQ}{dt} = -kA\frac{dT}{dx}$

Midterm 1 Equation Sheet

$Q = mc\Delta T = nC\Delta T$

$C_p - C_v = R = N_A k_B$

$\frac{dQ}{dt} = -kA\frac{dT}{dx}$

$W = \int P\, dV$

$PV = nRT = Nk_B T$

$e = \frac{W_{net}}{Q_{in}}$

$\Delta S = \int \frac{dQ}{T}$ (for reversible processes)

$dQ = T\, dS$

$\Delta S_{syst} + \Delta S_{env} > 0$

$\oint dS = 0$

FORCES + FIELDS

$F = qE = ma$

$F = k\frac{Q_1 Q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$ coulomb's law

* can use superposition for forces

$\vec{E} = \frac{\vec{F}}{q}$

$E = k\frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ ← for single point charge =

$E = \frac{\sigma}{\varepsilon_0}$ ← for 2 parallel plates

$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}$ ← for continuous charge dist

① Draw FBD for forces
② Apply Coulomb's law
③ Add vectorially + use vectors & symmetry

Electric Flux

$\Phi_E = EA\cos\theta$

$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

* use sphere or cylinder

$\oint E \cdot dA = E(4\pi r^2) = \frac{Q}{\varepsilon_0} \rightarrow E = \frac{Q}{4\pi r^2 \varepsilon_0}$

* for solid sphere use charge density

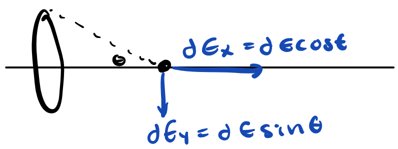
$\rho = \frac{\partial Q}{\partial V} \rightarrow Q_{enc} = \frac{\frac{4}{3}\pi r^3 \rho}{\frac{4}{3}\pi r_0^3 \rho} = \frac{r^3}{r_0^3} Q$

* or integrate to find charge enity

$Q_{enc} = \int_0^{r_0} \rho_E \, dV$

Example continuous charge problem

$\vec{E} = \int d\vec{E}$



rewrite charge (dq) and cos/sin
$\partial Q = \lambda \, d\ell$
$\cos\theta = \frac{x}{r}$
$r^2 = x^2 + a^2$

$\vec{E}_x = \int dE\cos\theta = \int \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta = \int \frac{\lambda \, d\ell}{4\pi\varepsilon_0 r^2}$

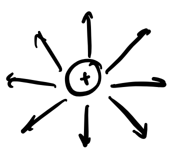
$= \frac{\lambda}{4\pi\varepsilon_0}\frac{x}{(x^2+a^2)^{3/2}}\int_0^{2\pi a} d\ell = \frac{\lambda x (2\pi a)}{4\pi\varepsilon_0 (x^2+a^2)^{3/2}}$

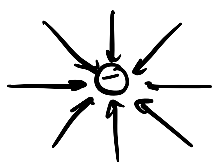
DENSITIES

$\lambda = \frac{Q}{\ell} \qquad \sigma = \frac{Q}{A} \qquad \rho = \frac{Q}{V}$

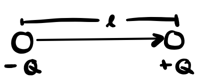
$Q = \int \lambda \, d\ell = \int \sigma \, dA = \int \rho \, dV$

OUT from ⊕ IN from ⊖





Dipoles

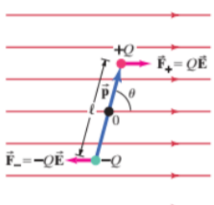


dipole moment: $\boxed{\vec{p} = Q\vec{\ell}}$

$\tau = \vec{p} \times \vec{E}$ ← want to turn dipole so $\vec{p}$ is parallel to $\vec{E}$

$= pE\sin\theta$

$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$



$U = -W = pE\cos\theta = -\vec{p}\cdot\vec{E}$

Electric potential

$W = qEd$

$U_b - U_a = -W$

$V_a = \frac{U_a}{q}$

$V_{ba} = V_b - V_a = \frac{U_b - U_a}{q}$

$V = -\int_a^b \vec{E}\cdot d\vec{\ell} = -E(x_f - x_i)$

equipotential lines are <u>PERPENDICULAR</u> to electric field at all points

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ ← single point charge

$\Delta U = qV_{ba}$

$\vec{E} = -\nabla V = -\underset{E_x}{\frac{\partial V}{\partial x}}\hat{x} - \underset{E_y}{\frac{\partial V}{\partial y}}\hat{y} - \underset{E_z}{\frac{\partial V}{\partial z}}\hat{z}$

capacitors

$C = \frac{Q}{V} \rightarrow Q = CV$

$C = \varepsilon_0\frac{A}{d} = K\varepsilon_0\frac{A}{d}$

parallel: $C_{eq} = C_1 + C_2 + \dots$

series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$

Energy: $U = \frac{1}{2}QV$

$= \frac{1}{2}CV^2$

$= \frac{1}{2}\frac{Q^2}{C}$

energy density: $u = \frac{1}{2}\varepsilon_0 E^2$

permittivity: $\varepsilon = K\varepsilon_0$

Circuits

$V = IR$

$R = \rho \frac{\ell}{A}$ $\sigma = \frac{1}{\rho}$ ← conductivity

$P = IV = I^2R = \frac{V^2}{R}$ $j = \frac{I}{A} = nqv_d$

$I = I_0 \sin\omega t$

$I_{rms} = \frac{I_0}{\sqrt{2}}$ $V_{rms} = \frac{V_0}{\sqrt{2}}$

Series: $R_{eq} = R_1 + R_2$

Parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Kirchoff's rules:
1. sum of all currents entering junction = currents leaving junction
2. sum of changes in potential around closed loop = 0

Batteries in series: ⊣|⊢⊣|⊢ ADD

⊣|⊢⊣⊢ SUBTRACT

RC circuits: $Q = Q_0(1 - e^{-t/RC})$

CHARGING $V_c = \mathcal{E}(1 - e^{-t/RC})$

$I = \frac{\mathcal{E}}{R} e^{-t/RC}$

$\tau = RC$
0.63 or 0.37

DISCHARGING (*no battery)

$Q = Q_0 e^{-t/RC}$

$V_c = V_0 e^{-t/RC}$

$I = I_0 e^{-t/RC}$

Hall Effect = generation of voltage difference ⊥ to flow of current in conductor placed in magnetic field

* sideways deflection from Lorentz force → ⊥ to I & B
charges accumulate on one side

Hall Field $E_H = ev_d B$

Hall EMF $\mathcal{E}_H = E_H d = v_d B d$

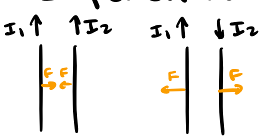
Drift speed $v_d = \frac{\mathcal{E}_H}{Bd}$

d = width of conductor

Magnetic Fields

Straight wire: $B = \frac{\mu_0 I}{2\pi r}$

2 parallel wires force

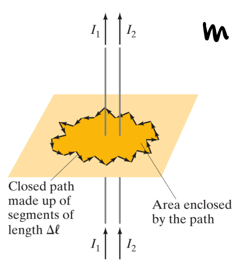


$F = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} \ell$

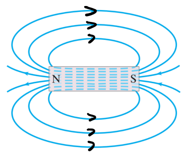
Ampere

$\frac{F}{\ell} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} = 2\times10^{-7}$ N/m

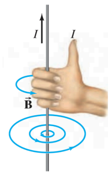
Ampere's Law: $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl}$



Magnetism...



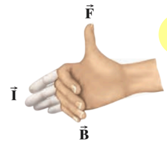
line point N → S



current produces mag field

Thumb = current

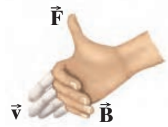
fingers = mag field



$\vec{F} = I\vec{\ell} \times \vec{B} = I\ell B\sin\theta$

$F_{max} = I\ell B$
↳ current perpendicular to field

moving electric charge in mag field



↑ pos charge particles

$\vec{F} = q\vec{v} \times \vec{B} = qvB\sin\theta$

$F_{max} = qvB$ when $\vec{v} \perp \vec{B}$

* FOR NEGATIVE charges flip F sign

* centripetal acceleration

$qvB = \frac{mv^2}{r}$ time $T = 2\pi\frac{r}{v}$ freq $f = \frac{1}{T}$

cathode ray

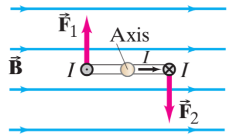
electrons

$evB = m\frac{v^2}{r}$

$\frac{e}{m} = \frac{v}{Br} = \frac{E}{B^2 r}$

Lorentz Eq: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ * E field & B field

Torque



2 torques on each lever arm

$F = IaB \rightarrow \tau = Iab\frac{b}{2} + Iab\frac{b}{2}$

$= I\underbrace{ab}_{A}B = IAB$

τ w/ n coils $=$ $NIAB\sin\theta$ $= NI\vec{A} \times \vec{B}$

Magnetic dipole moment: $\vec{\mu} = NI\vec{A}$

↳ $\tau = \vec{\mu} \times \vec{B}$

Potential energy: $U = -\vec{\mu} \times \vec{B}$

Solenoid = long coil w/ many loops

$B = \mu_0 n I$ $n = \frac{N}{\ell}$ = # of loops per unit length

Toroid = solenoid bent into circle

$\boxed{B = \frac{\mu_0 NI}{2\pi r}}$

Biot-Savart Law: (non symmetrical situations)

$\boxed{d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{r}}{r^2}}$ $\hat{r} = \frac{\vec{r}}{r}$ unit vector in dir of r

magnitude: $dB = \frac{\mu_0 I d\ell \sin\theta}{4\pi r^2}$

Total: $\boxed{\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{\ell} \times \hat{r}}{r^2}}$

↓ only current element B

INDUCTION

changing magnetic field induces EMF

magnetic flux: $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$

$\Phi_B = \int \vec{B}\cdot d\vec{A}$

Induced EMF: $\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$

N loops: $\mathcal{E} = -N\frac{d\Phi_B}{dt}$

* current produced by induced emf creates magnetic field that OPPOSES the original change in mag flux

3 ways to induce EMF:
1. changing magnetic field
2. changing area A of loop in field
3. changing loop's orientation θ

EMF in moving conductor

$\mathcal{E} = \frac{d\Phi_B}{dt} = B\ell v$

$W = qvB\ell$

Electric generators

mechanical → electric energy

$\mathcal{E} = BA\omega\sin\omega t$

$= NBA\omega\sin\omega t$

$= \mathcal{E}_0\sin\omega t$

$\mathcal{E}_{rms} = \frac{NB\omega A}{\sqrt{\ }} = \frac{\mathcal{E}_0}{\sqrt{2}}$

* freq $f = \frac{\omega}{2\pi}$

Transformers

- primary & secondary coils

$V_S = N_S\frac{d\Phi_B}{dt}$ | $V_P = N_P\frac{d\Phi_B}{dt}$

$\frac{V_S}{V_P} = \frac{N_S}{N_P}$

$N_S > N_P \rightarrow \boxed{V_S > V_P = \text{step up}}$

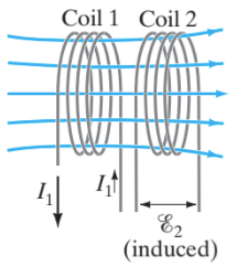
$N_S < N_P \rightarrow \boxed{V_S < V_P = \text{step down}}$

$\frac{I_S}{I_P} = \frac{N_P}{N_S}$

Magnetic Flux → Electric Field

$\boxed{\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}}$

Mutual inductance



$M_{21} = \frac{N_2\Phi_{21}}{I_1}$

$\mathcal{E}_2 = -N_2\frac{d\Phi_1}{dt} = -M_{21}\frac{dI_1}{dt}$

$$\boxed{\begin{array}{l} M = M_{12} = M_{21} \\ \mathcal{E}_1 = -M\frac{dI_2}{dt} \qquad \mathcal{E}_2 = -M\frac{dI_1}{dt} \end{array}}$$

Self inductance

$\boxed{L = \frac{N\Phi_B}{dt}} \rightarrow \boxed{\mathcal{E} = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}}$

$W = \frac{1}{2}LI^2$

$U = \frac{1}{2}LI^2 = \frac{1}{2}\frac{B^2}{\mu_0}A\ell$

energy density $u = \frac{1}{2}\frac{B^2}{\mu_0}$

RL circuits

GROWTH

$\boxed{I = \frac{V_0}{R}\left(1 - e^{-t/\tau}\right) \qquad \tau = \frac{L}{R}}$

DECAY

$\boxed{I = I_0 e^{-t/\tau}}$

LC circuits

$Q = Q_0\cos(\omega t + \phi)$ $\quad \omega = 2\pi f = \sqrt{\frac{1}{LC}}$

$I = I_0\sin(\omega t + \phi)$ $\quad$ max current: $I_0 = \omega Q_0 = \frac{Q_0}{\sqrt{LC}}$

energy in electric field $\quad U_E = \frac{1}{2}\frac{Q^2}{C} = \frac{Q_0^2}{2C}\cos^2(\omega t + \phi)$

energy in magnetic field $\quad U_B = \frac{1}{2}LI^2 = \frac{Q_0^2}{2C}\sin^2(\omega t + \phi)$

Total energy: $U = U_E + U_B = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2 = \frac{Q_0^2}{2C}$

RLC circuit

$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0$

$R^2 < \frac{4L}{C}$ $\quad$ underdamped, oscillating

$R^2 > \frac{4L}{C}$ $\quad$ overdamped