Gaussian Elimination

3 operations
1. Multiplying a row by a nonzero scalar
2. swapping rows
3. Adding a scalar multiple of a row to another row

Reduced row echelon form $\left[\begin{array}{cccc|c} 1 & 0 & * & 0 & * \\ 0 & 1 & * & 0 & * \\ 0 & 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

Linear Equations
1. superposition: If $x+y=z \rightarrow f(y+z) = f(y)+f(z)$
2. Homogeneity: $f(\alpha x) = \alpha f(x)$

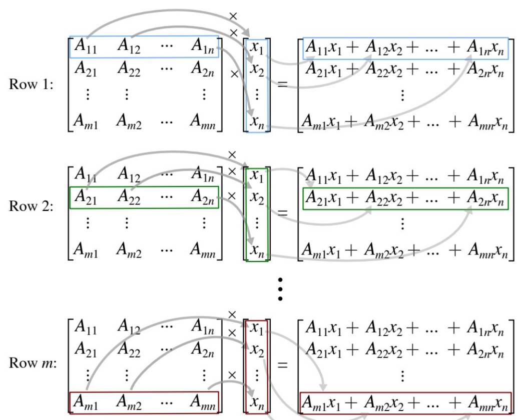
$f(x) = b^2x$ $\quad \left.\begin{array}{l} f(ax) = b^2ax \\ af(x) = b^2ax \end{array}\right] \quad b^2ax = b^2ax$ ✓ homogeneity

$\left.\begin{array}{l} f(y+z) = b^2(y+z) \\ f(y)+f(z) = b^2y + b^2z \end{array}\right] =$ ✓ superposition

Vectors / Matrices

$$\vec{y}^T\vec{x} = \begin{bmatrix} y_1 & y_2 & \cdots & y_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = y_1x_1 + y_2x_2 + \cdots + y_nx_n$$

$\left.\begin{array}{l} \vec{x}+\vec{y} = \vec{y}+\vec{x} \\ (\vec{x}+\vec{y})+\vec{z} = \vec{x}+(\vec{y}+\vec{z}) \\ \vec{x}+\vec{0} = \vec{x} \\ \vec{x}+(-\vec{x}) = \vec{0} \end{array}\right]$ Addition



$\left.\begin{array}{l} (\alpha\beta)\vec{x} = \alpha(\beta\vec{x}) \\ (\alpha+\beta)\vec{x} = \alpha\vec{x}+\beta\vec{x} \\ 1\vec{x} = \vec{x} \end{array}\right]$ scalar multiplication

$\vec{x} = \begin{bmatrix} x^1 \\ \vdots \\ x_n \end{bmatrix}$ then $\vec{x}^T = [x^1 \cdots x_n]$

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Linear Transformation: $f_A(\vec{x}) = A\vec{x}$

$\left.\begin{array}{l} \text{ccw} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \\ \text{cw} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \end{array}\right]$ Rotation

x-axis $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ y-axis $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$y=x$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $y=-x$ $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Reflect (R) then rotate (O)
$R\vec{v}$ then multiply w $O \rightarrow O(R\vec{v})$

Linear (in)dependence

$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Linear dependence: If vector can be written as combo of other vectors
$a_1\vec{v}_1 + \dots + a_n\vec{v}_n = \vec{0}$ and not all α_i's $= 0$

independence: if $\alpha_1\vec{v}_1 + \dots + \alpha_n\vec{v}_n = \vec{0}$ implies $\alpha_1 = \dots = \alpha_n = 0$

Span: set of all linear combinations of $\{v_1 \dots v_n\}$ $\quad$ span $\left\{ \sum_{i=1}^{n} a_i\vec{v}_i \,\middle|\, \alpha_i \in \mathbb{R} \right\}$
↳ range / column space
* can solve w/ Gaussian Elimination

Proofs

☆ Review proof examples

1. Write down what you know (rephrasing or in math)
2. Write down what you want to show (map out your path)
3. Find similarities (how can I form look like the other)
4. Try a simple example for intuition
5. Manipulate both sides of claim & JUSTIFY each step

State Transition Matrices / Inverses

$$\begin{bmatrix} P_{A\to A} & P_{B\to A} & P_{C\to A} \\ P_{A\to B} & P_{B\to B} & P_{C\to B} \\ P_{A\to C} & P_{B\to C} & P_{C\to C} \end{bmatrix}$$

Given current state : $\vec{v}[t]$

state transition matrix : A

$$\vec{v}[t+1] = A\vec{v}[t]$$

A square matrix M and its inverse M^{-1} satisfies $MM^{-1} = I$

Let $M = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ and $M^{-1} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$\hookrightarrow \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$[M \mid I_n] \rightarrow [I_n \mid M^{-1}]$$

$\hookrightarrow \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right]$ ☆ use Gaussian Elimination

$$M^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

A is invertible $\longleftrightarrow$ equation $A\vec{x} = \vec{b}$ has a unique solution

A is invertible $\longleftrightarrow$ A has linearly independent columns

$AB = BA = I$

Vector Spaces

☆ review problem solving techniques

vector space $\mathbb{V}$ is a set of vectors that satisfies $\begin{cases} \text{vector addition} \\ \text{scalar multiplication} \end{cases}$ see properties $\longrightarrow$

Basis = series of vectors that defines a vector space $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$

1. $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ must be linearly independent
2. For any vector $\vec{v} \in \mathbb{V}$, $\vec{v} = \alpha_1\vec{v}_1 + \alpha_2\vec{v}_2 + \dots + \alpha_n\vec{v}_n$ (scalars)

☆ minimum set of vectors needed to represent vector space

Dimension = # of basis vectors

Subspace $\mathbb{U}$ = subset of vector space $\mathbb{V}$

1. contains the 0 vector : $\vec{0} \in \mathbb{U}$
2. closed under vector addition : $\vec{v}_1, \vec{v}_2 \in \mathbb{U} \rightarrow \vec{v}_1 + \vec{v}_2$ must be in $\mathbb{U}$
3. closed under scalar multiplication : $\vec{v} \in \mathbb{U}$ & scalar $\alpha \in \mathbb{R}$, $\alpha\vec{v}$ in $\mathbb{U}$

$$\text{Col}(A) = \{\vec{v} \mid \vec{v} = \sum_{i=1}^{m} x_i \vec{a}_i\} \text{ where } x_i\text{'s are scalars}$$

☆ To solve $\rightarrow$ use Gaussian elimination and columns w/ pivots = vectors in the span

$D = \begin{bmatrix} 1 & -1 & -3 & 4 \\ 3 & -3 & -5 & 8 \\ 1 & -1 & -1 & 2 \end{bmatrix}$

(2) − 3(1) $\begin{bmatrix} 1 & -1 & -3 & 4 \\ 0 & 0 & 4 & -4 \\ 1 & -1 & -1 & 2 \end{bmatrix}$

(3) − (1) $\begin{bmatrix} 1 & -1 & -3 & 4 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & 2 & -2 \end{bmatrix}$

/4, /2 $\begin{bmatrix} 1 & -1 & -3 & 4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 & -3 & 4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\boxed{\text{col}(D) = \text{span}\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -5 \\ -1 \end{bmatrix} \right\} \quad \text{Dim} = 2}$

$\text{Rank}(A) = \dim(\text{col}(A)) \leq \min(m,n)$ (# of linearly independent cols)

Nullspace: set of vectors mapped to 0 by A $\rightarrow \{\vec{x} \mid A\vec{x} = \vec{0}, \vec{x} \in \mathbb{R}^n\}$ $\boxed{A\vec{x} = \vec{0}}$

☆ use GA to set matrix = 0 $\rightarrow$ create an $\vec{x}$ for $x_1 \cdots x_n$ in matrix

$\rightarrow$ scalar multiplication

$$\left[\begin{array}{cccc|c} 1 & -1 & -3 & 4 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$(1)+3(2) \left[\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} x_1 = x_2 - x_4 \\ x_3 = x_4 \end{array} \quad \text{Let } \begin{array}{l} x_2 = \alpha \\ x_4 = \beta \end{array}$$

$$\vec{x} = \begin{bmatrix} \alpha - \beta \\ \alpha \\ \beta \\ \beta \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\boxed{\text{Null}(D) = \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\} \quad \text{Dim} = 2}$$

$\text{col}(A)$ = # of linearly independent cols

$\text{Null}(A)$ = # of linearly dependent cols

$$\underbrace{m - \dim(\text{col}(A)) = \dim(N(A))}_{\text{Rank-nullity theorem}}$$

NON-TRIVIAL: columns = linearly dependent

TRIVIAL: INDEPENDENT $\rightarrow \vec{0}$

Eigenvector / Eigenvalues

Eigenvector: nonzero vector $A\vec{x} = \lambda\vec{x}$ where λ is eigenvalue of $\vec{x}$

$A\vec{x} = \lambda\vec{x} = \vec{0}$

$(A - \lambda I_n)\vec{x} = \vec{0} \longrightarrow \det(A - \lambda I) = 0$

☆ All eigenvectors w/ diff eigenvalues are linearly indep

$$\underbrace{\lambda^2 - (a+d)\lambda + (ad-bc)}_{\text{characteristic polynomial}} = 0$$

2 distinct real eigenvals $\rightarrow$ linearly independent eigenvectors

Steady state freq $\longrightarrow$ eigenvector associated with $\lambda = 1$ and normalize so that the columns sum to 1

☆ review last dis

$\vec{x}[t] = \alpha_1(\lambda_1^t \vec{v}_1) + \alpha_2(\lambda_2^t \vec{v}_2) + \ldots + \alpha_n(\lambda_n^t \vec{v}_n)$ ☆ want to know if $\vec{x}[t]$ will converge

1. If $|\lambda_i| > 1$ then $\lambda_i^t \vec{v}_i \longrightarrow \infty$
2. If $\lambda_i = -1$ then $\lambda_i^t \vec{v}_i$ will oscillate

] fail to converge

3. If all λ_i $-1 < \lambda_i \leq 1$ then each term $\rightarrow 0$ $(\lambda_i \neq 1)$ or stay the same $(\lambda_i = 1)$
   $\hookrightarrow \vec{x}[t]$ will always converge to a fixed value

Properties

For a square matrix A:
1. A is invertible
2. $A\vec{x} = \vec{b}$ has a unique soln for any $\vec{b}$
3. A has linearly independent cols
4. A has a trivial null space
5. Determinant of $A \neq 0$

$|A \cdot B| = |A| \cdot |B|$ $\quad$ λ of $A \neq 0$

Matrix & transpose have same determinants

$$\lim_{n \to \infty} \vec{x}[n] = A^n \vec{x}[0]$$
$$= A^n[\alpha_1\vec{v}_1 + \alpha_2\vec{v}_2 + \ldots + \alpha_n\vec{v}_n]$$
$$= \alpha_1(A^n\vec{v}_1) + \alpha_2(A^n\vec{v}_2) + \ldots + \alpha_n(A^n\vec{v}_n)$$
$$= \alpha_1(\lambda_1^n\vec{v}_1) + \alpha_2(\lambda_2^n\vec{v}_2) + \ldots$$

☆ If invertible matrix A has eigenvalue $\lambda \rightarrow$ then A^{-1} has eigenvalue $\frac{1}{\lambda}$

☆ All vectors in the nullspace of a matrix are in its eigenspace for $\lambda = 0$ $\quad A\vec{x} = \vec{0} \rightarrow A\vec{x} = 0\vec{x}$

* REVIEW HW PROOFS
* even if $\lambda > 1$ → doesn't necessarily mean it'll diverge
* look at equivalent definitions + expand stuff out

$G = M_1 M_2$ (Alr know M_1 and M_2 have inverses bc linear indep)

$M_1^{-1} G = M_1^{-1} M_1 M_2$

$M_1^{-1} G = M_2$

$M_2^{-1} M_1^{-1} G = M_2^{-1} M_2$

$\underbrace{M_2^{-1} M_1^{-1}}_{G^{-1}} G = I$

* $AB = I$

Then $B = A^{-1}$

$A = B^{-1}$

$14 \times 5 = 70$ $\quad 14 \times 14 = 56 + 140 = 196$ $\quad 70 - 25 = 45$ $\quad 196 - 45 = 151$

$\lambda_1 = \frac{5}{2}$ $\quad (a - \frac{5}{2})(a - \frac{5}{2}) - b^2 = 0$

$a^2 - 5a + \frac{25}{4} - b^2 = 0$

$\lambda_2 = \frac{9}{2}$ $\quad (a - \frac{9}{2})(a - \frac{9}{2}) - b^2 = 0$

$a^2 - 9a + \frac{81}{4} - b^2 = 0$

$\left(\frac{14}{4}\right)^2 - \frac{70}{4} + \frac{25}{4} - b^2 = 0$

$\frac{196}{4} - \frac{45}{4} = b^2$

$\sqrt{\frac{151}{4}} = \sqrt{b^2}$

$\cancel{a^2} - 5a + \frac{25}{4} - \cancel{b^2} = \cancel{a^2} - 9a + \frac{81}{4} - \cancel{b^2}$

$81 - 25 = 56$

$4a = \frac{81}{4} - \frac{25}{4}$

$4a = \frac{56}{4}$

$4a = 14$

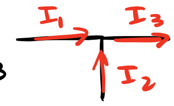
$a = 14/4$

KVL & KCL & Circuit Elements

Wire: $V = 0$
Resistor: $V = IR$
Open circuit: $I = 0$

Voltage source: $V = V_s$
Current source: $I = I_s$

KCL: current flowing into node = current flowing out of node : $I_1 + I_2 = I_3$



KVL: $\sum_{\text{loop}} V_k = 0$

* start from pos end

* if $+ \to -$, subtract voltage
$- \to +$, add voltage

Power: $P = IV = \frac{V^2}{R} = I^2R$ (watts)

Ohm's law: $V_{elem} = I_{elem} R$

current: $I = \frac{\partial Q}{\partial t}$

Resistivity: $R = \rho \times \frac{L}{A}$

Circuit Analysis / NVA

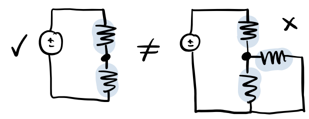
- Passive sign convention : enter positive
exit negative

* node = region of circuit w/ same voltage throughout

1. Pick reference node → label 0V
2. label other nodes
3. label currents
4. Add +/− labels on non-wire elements
5. Use KCL to write eqs at labeled nodes
6. Write I-V relationship
7. Solve w/ substitution

Voltage Divider

* Make sure currents equal through resistors

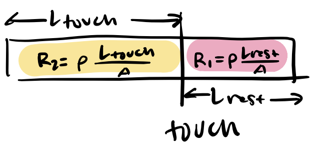


$$U_{mid} = \frac{R_2}{R_1 + R_2} V_s$$

parallel : $R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$

series: $R_{eq} = R_1 + R_2$

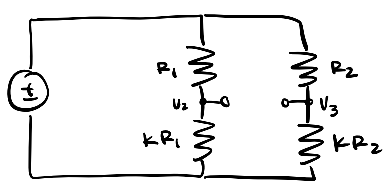
Resistive Touchscreens



$$U_{mid} = \frac{L_{touch}}{L} V_s$$

$P = IV$ $+P \to$ power dissipated
$-P \to$ power delivered

* IDEAL AMMETER = wire
IDEAL VOLTMETER = open-circuit (∞ resistance)

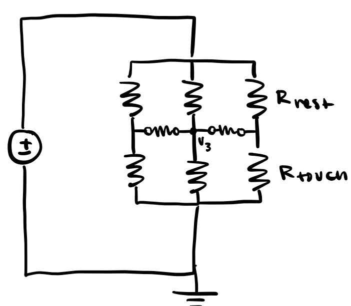


2 voltage dividers

$$V_2 = V_3 = \frac{k}{1+k} V_s$$

* Resistor proportions equal, so same voltage drop

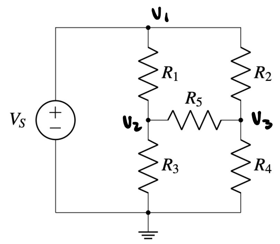
2D touchscreen



$R_{rest} = R_{touch}$ so $U_2 = V_3 = U_4$ → replace horizontal resistors w open circuits

$$U_3 = \frac{L_{touch}}{L} \times V_s \quad \text{where } L_{touch} = L_{touch,vertical}$$

$$U_3 = \frac{R_{touch}}{R_{touch} + R_{rest}} \times V_s \quad \text{where } R_{touch} = \rho \frac{L_{touch,horizontal}}{A}$$



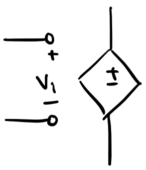
① $V_1 = V_S$

② KCL @ unknown nodes & current flowing out of node

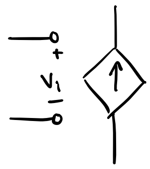
$$\frac{V_2 - V_S}{R_1} + \frac{V_2 - V_3}{R_5} + \frac{V_2}{R_3} = 0 \qquad \frac{V_3 - V_S}{R_2} + \frac{V_3}{R_4} + \frac{V_3 - V_2}{R_5} = 0$$

Superposition + Equivalence

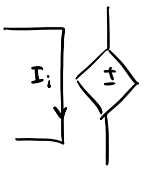
① Null one source
② Write KCL eqs
③ Use N relations & NVA
④ Combine final results



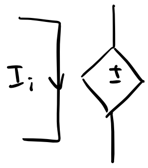
αV_i : Voltage-controlled voltage source



gV_i : Voltage-controlled current source

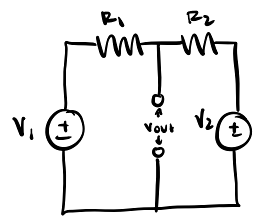


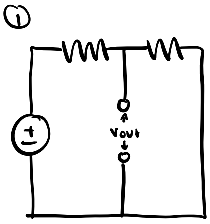
rI_i : current-controlled voltage source



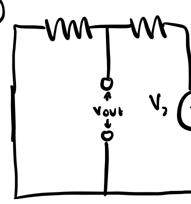
βI_i : current controlled current source

Superposition
- For each indep source
  - set other sources = 0
    - V source: replace w/ wire
    - C source: replace w/ open circuit
  - Compute voltages + currents
  - V_{out} = sum of $V_{out,k}$ for all k



① 

$$V_{out_1} = \frac{R_2}{R_1 + R_2} V_1$$

② 

$$V_{out_2} = \frac{R_1}{R_1 + R_2} V_2$$

$$V_{out} = \frac{R_2}{R_1 + R_2} V_1 + \frac{R_1}{R_1 + R_2} V_2$$

✱ 2 circuits are equivalent if they have same I-V relationship

THEVENIN & NORTON EQUIVALENT

✱ can be used to calculate power in un-transformed elements

Ⓐ Find thevenin voltage
① ID all nodes in circuit
② $V_{th} = V_A - V_B$
✱ use nodal analysis or superposition

✱ Thevenin voltage = open circuit voltage between terminals A & B

Ⓑ Find norton current
① connect a wire through A & B (short circuit)
② Simplify circuit by removing resistors (path of least resistance)
③ $I_N = I_{AB}$

✱ Norton current = current between terminals A & B

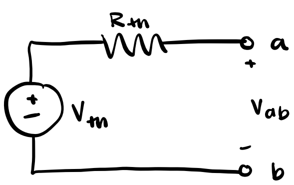
✱ $R_{eq} = \frac{V_{th}}{I_N}$

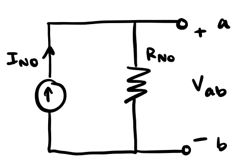
Thevenin: $V_L = \frac{V_{th} R_L}{R_{eq} + R_L}$ Norton: $I_L = \frac{I_N R_{eq}}{R_{eq} + R_L}$

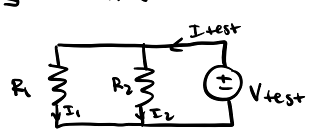
Ⓒ Find Thevenin/Norton R_{eq}
① Turn off all independent sources in circuit
② Apply test voltage V_{test} → calculate I_{test} that flows through test voltage source

$$R_{eq} = \frac{V_{test}}{I_{test}}$$

example







$I_1 = \frac{V_{test}}{R_1}$ $I_2 = \frac{V_{test}}{R_2}$

$$I_{test} = I_1 + I_2 = \frac{V_{test}}{R_1} + \frac{V_{test}}{R_2}$$

Capacitors

☆ stores charge

$$Q = CV_c \qquad I = C\frac{dV_c}{dt}$$

☆ current only if voltage changing with time

$I = C\frac{dV_c}{dt}$

$I\,dt = C\,dV_c$

$\int_0^t I\,dt = \int_0^{V_c} C\,dV_c$

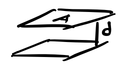
$V_c(t) = \frac{I}{C}t + V_c(0)$

$V_c(t) = \frac{I}{C}t + V_c(0)$

Parallel: $C_{eq} = C_1 + C_2$

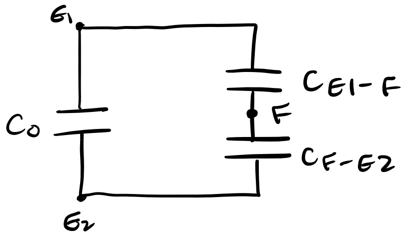
Series: $C_{eq} = C_1 \| C_2 = \frac{C_1C_2}{C_1+C_2}$

$C = \varepsilon\frac{A}{d}$

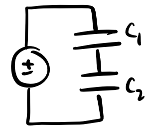


$E = \frac{1}{2}CV^2$

Capacitor w/ touch

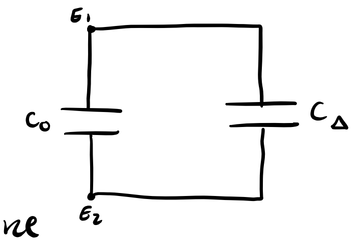


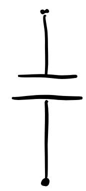
$\boxed{\text{☆ STEADY STATE = NO CURRENT}}$



$V_{C_2} = V_s\left(\frac{C_1}{C_1+C_2}\right)$

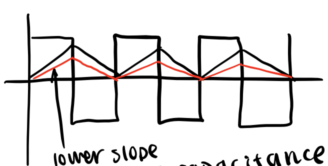
combine C_{E1-F} and C_{F-E2} to $C\Delta$





$C_{eq,E1-E2} = C_0 + C\Delta$

☆ Apply periodic current source

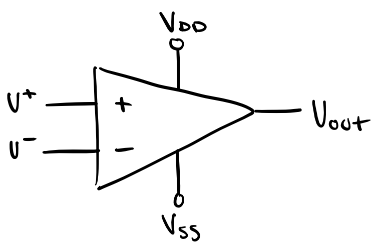


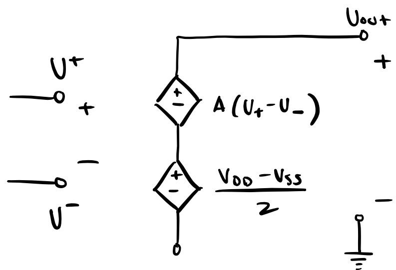
lower slope w/ larger capacitance so this is w/ touch

$$V_c(t) = \begin{cases} \frac{I_1}{C}t & 0 \le t \le \frac{T}{2} \\ -\frac{I_1}{C}\left(t - \frac{T}{2}\right) + \frac{I_1T}{2C} & \frac{T}{2} < t \le T \end{cases}$$

$$\boxed{V_c(t) = \frac{I}{C}t + V_c(0)}$$

Op-Amps

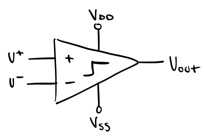




$A = \infty$

IDEAL

comparator



$$V_{out} = \begin{cases} V_{ss} \\ A(V_+ - V_-) + \frac{V_{DD}+V_{ss}}{2} \\ V_{DD} \end{cases}$$

$A(V_+ - V_-) + \frac{V_{DD}+V_{ss}}{2} < V_{ss}$

$V_{ss} \le A(V_+ - V_-) + \frac{V_{DD}+V_{ss}}{2} \le V_{DD}$

$V_{DD} < A(V_+ - V_-) + \frac{V_{DD}+V_{ss}}{2}$

☆ $V^+ < V^-$ $\quad V_{out} = V_{ss}$

$V^+ > V^-$ $\quad V_{out} = V_{DD}$

Charge Sharing

☆ Floating node where charge can't flow in/out

☆ Voltage drops from plate w/ positive charge to plate holding negative charge

1. Label voltages across all capacitors
2. Draw circuit in each phase
3. ID all floating nodes during phase 2
4. Examine each floating node individually
   - a) ID capacitor plates connected to that node phase 2
   - b) calculate charge on those plates phase 1
5. Find total charge on floating node in phase 2
6. Charge in steady state phase 1 = charge in phase 2

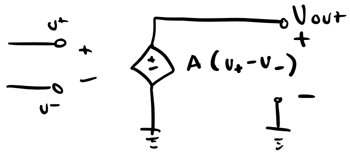
☆ DON'T use parallel/series eq cap

☆ Floating node $\varnothing_2$ not always floating $\varnothing_1$

☆ use node voltages according to labelled polarities

NEG FEEDBACK

some function of output fed back to input to keep output at some finite value

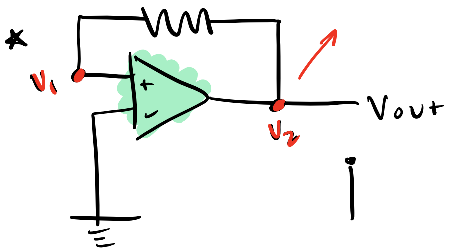


GOLDEN RULES

① $I_+ = I^- = 0$ ← even w/o neg feedback

② $v^+ = v^-$

*



$v_2 \uparrow$ then $v_1 \uparrow$ so $v^+ \uparrow$ and since $V_{out} = A \cdot (v^+ - v^-)$

so since $v^+ \uparrow$ then $V_{out} \uparrow$

POSITIVE FEEDBACK!

↳ flip polarities

$v_2 \uparrow$

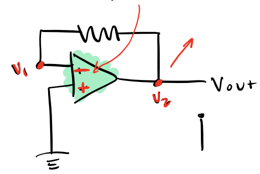
$v_1 \uparrow$

$v^- \uparrow$

$V_{out} = A(v^+ - v^-)$

$V_{out} \downarrow$

NEG FEEDBACK



① GR2 → $\cancel{v^+} = v^-$ GND 0

$v^- = v^+ = 0$

② GR1 → $I_1 + I_2 = \cancel{I^-} = 0$

& NOW ADD KCL! & APPLY VOLTAGE DEFS

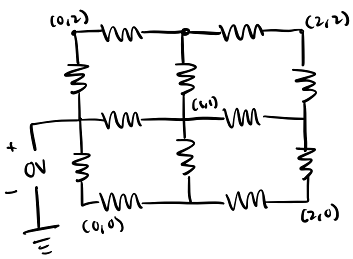
$\frac{\overset{0}{\cancel{v_2}} - v_1}{R_1} + \frac{v_3 - \overset{0}{\cancel{v_2}}}{R_2} = 0$

$-\frac{\overset{V_{in}}{\cancel{v_1}}}{R_1} + \frac{\overset{V_{out}}{\cancel{v_3}}}{R_2} = 0$

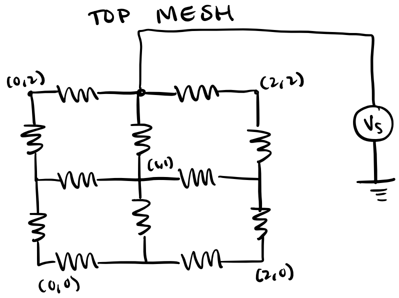
$-\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} = 0$

2D Touchscreen

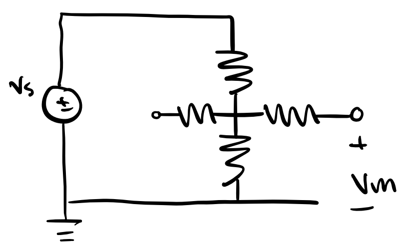
BOTTOM MESH



TOP MESH



GENERAL CIRCUIT



Design Procedure

1. Restate goals of circuit
2. strategy: what you can measure, how/what you need to change (* use block diagrams)
3. Implement: use blocks & think abt how they can be modified / extended
4. verify & check block-to-block connections (check contradictions)

* think about which elements depend on which aspects

Elements: op-amps, resistors, capacitors, comparators, switches

* Use buffer to connect parts
* Gain : $A_v = \frac{\text{output voltage}}{\text{input voltage}}$

Inner Product + Norms

$\boxed{\langle \vec{x}, \vec{y} \rangle} = \vec{x} \cdot \vec{y} = \boxed{\vec{x}^T \vec{y}} = [x_1 \ x_2 \cdots x_n] \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = x_1y_1 + x_2y_2 + \cdots + x_ny_n$

$= \boxed{\|x\| \|y\| \cos\theta}$

1. symmetry: $\langle \vec{v}, \vec{v} \rangle = \langle \vec{v}, \vec{v} \rangle$
2. linearity: $\langle \alpha\vec{v}, \vec{v} \rangle = \alpha \langle \vec{v}, \vec{v} \rangle$

   $\langle \vec{v} + \vec{w}, \vec{v} \rangle = \langle \vec{v}, \vec{v} \rangle + \langle \vec{w}, \vec{v} \rangle$
3. positive-definiteness: $\langle \vec{v}, \vec{v} \rangle \geq 0$

orthogonal: when inner product = 0

NORM

Euclidean norm: $\|\vec{x}\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2} = \sqrt{\langle \vec{x}, \vec{x} \rangle}$ (magnitude of vector)

$\|\vec{x}\| = 0$ iff $\vec{x} = \vec{0}$

$\|a\vec{x}\| = |a| \|\vec{x}\|$ $\quad \|\vec{x} + \vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|$

$\hat{x} = \frac{\vec{x}}{\|x\|}$ $\quad \hat{y} = \frac{\vec{y}}{\|y\|}$

$\langle \vec{x}, \vec{x} \rangle = \|\vec{x}\|^2 = \vec{x}^T \vec{x}$

$|\langle \vec{x}, \vec{y} \rangle| \leq \|\vec{x}\| \|\vec{y}\|$

projection of $\vec{x}$ onto $\vec{y}$: $\text{proj}_{\vec{y}} \vec{x} = \frac{\langle \vec{y}, \vec{x} \rangle}{\|\vec{y}\|^2} \vec{y}$

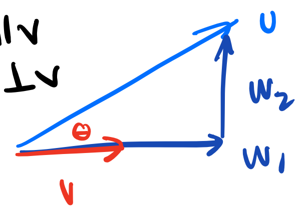
$\vec{e} = \vec{x} - \text{proj}_{\vec{y}} \vec{x} \longrightarrow \langle \vec{e}, \vec{y} \rangle = \langle \vec{x} - \text{proj}_{\vec{y}}\vec{x}, \vec{y} \rangle = \langle \vec{x}, \vec{y} \rangle - \langle \vec{y}, \vec{x} \rangle = 0 \longrightarrow \vec{e}$ & $\vec{y}$ are orthogonal

$\vec{x} \in \mathbb{R}^2$ such that projection of $\vec{b}$ onto col space A is $A\vec{x}$ where x from least squares

* vector $\text{proj}_{\vec{y}} \vec{x}$ is vector in span $\{\vec{y}\}$ that is closest to $\vec{x}$
* BIG inner product = MORE similar

$w_1 \parallel v$

$w_2 \perp v$



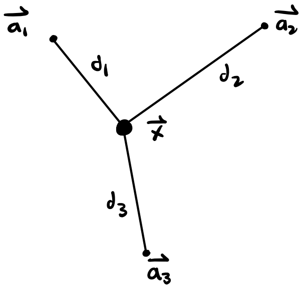
component of u that travels along v (proj of u onto v)

$w_1 = \text{proj}_v u$

$w_2 = u - w_1 = u - \text{proj}_v u$

orthogonal to v

Trilateration



$\|\vec{x} - \vec{a}_1\|^2 = d_1^2$

$\|\vec{x} - \vec{a}_2\|^2 = d_2^2$

$\|\vec{x} - \vec{a}_3\|^2 = d_3^2$

$(x - a_1)^T (x - a_1) = \vec{x}^T \vec{x} - 2\vec{a}_1^T \vec{x} + \|\vec{a}_1\|^2 = d_1^2$

$\vec{x}^T \vec{x} - 2\vec{a}_2^T \vec{x} + \|\vec{a}_2\|^2 = d_2^2$

$\vec{x}^T \vec{x} - 2\vec{a}_3^T \vec{x} + \|\vec{a}_3\|^2 = d_3^2$

$$\begin{bmatrix} 2(\vec{a}_1 - \vec{a}_2)^T \\ 2(\vec{a}_1 - \vec{a}_3)^T \end{bmatrix} \vec{x} = \begin{bmatrix} \|\vec{a}_1\|^2 - \|\vec{a}_2\|^2 - d_1^2 + d_2^2 \\ \|\vec{a}_1\|^2 - \|\vec{a}_3\|^2 - d_1^2 + d_3^2 \end{bmatrix}$$

* subtract eqs to get rid of the quadratic term

Cross-correlation

* measure of similarity based on inner product

$d = v\tau \qquad \tau = \text{argmax}(\text{circcorr}(\vec{r}, \vec{s}) [k])$

$\text{corr}_{\vec{x}}(\vec{y}) [k] = \sum_{i=-\infty}^{\infty} x[i] y[i-k]$ linear

* received signal = same / other signal shifts
* use zero padding
* length-1 shifts to right & left

$\text{circcorr}(\vec{x}, \vec{y}) [k] = \sum_{i=0}^{N-1} x[i] y[(i-k)_N]$ circular

$\text{corr}_N(\vec{x}, \vec{y}) [k] = \sum_{i=0}^{N-1} x[i] y[i-k]$ periodic

* assume signal repeating w/ per N

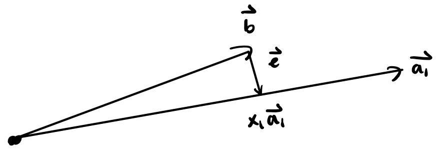
autocorrelation: correlation between signal & itself : $\text{corr}_x \vec{x}$

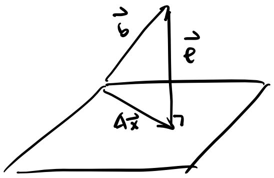
$\text{corr}_y(\vec{x}) \neq \text{corr}_x(\vec{y})$

Least squares

$A\vec{x} = \vec{b}$ where $\boxed{\text{more}}$ equations than unknowns

$\|\vec{e}\| = \|\vec{b} - x_1\vec{a}_1\|$ $\qquad \langle \vec{e}, x_1\vec{a}_1 \rangle = 0$





$$x_1 = \frac{\langle \vec{b}, \vec{a}_1 \rangle}{\langle \vec{a}_1, \vec{a}_1 \rangle}$$

$\langle \vec{e}, \vec{a}_i \rangle = 0 \longleftrightarrow \vec{a}_i^T \vec{e} = 0 \rightarrow A^T\vec{e} = \vec{0}$

$\underset{\text{actual}}{\|\vec{e}\| = \|\vec{b}} - \underset{\text{predicted}}{A\vec{x}\|}$ ← we want to minimize e

- NEED $\boxed{\text{linearly indep}}$ columns
- rows $\geq$ cols
- careful about actually applying model

$$\boxed{\vec{x} = (A^TA)^{-1}A^T\vec{b}}$$

$e = \vec{b} - A\vec{x}$ is orthogonal to cols of A

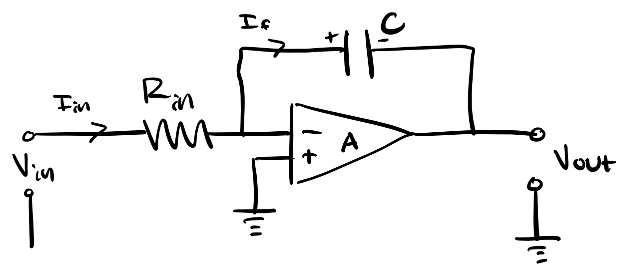
$(A^T)(\vec{b} - A\vec{x}) = A^T(\vec{b} - A(A^TA)^{-1}A^T\vec{b})$
$= A^T\vec{b} - A^TA(A^TA)^{-1}A^T\vec{b}$
$= A^T\vec{b} - IA^T\vec{b}$
$= A^T\vec{b} - A^T\vec{b} = \vec{0}$

If vector is orthogonal to col(A) it's in Null(A^T)

$\langle \vec{e}, \vec{a_i} \rangle = 0 \longleftrightarrow \vec{a_i}^T\vec{e} = 0$

↑ cols of A

Integrator Circuit



- Golden rules:

$I_{in}(t) = \frac{V_{in}(t) - 0}{R_{in}} = \frac{V_{in}(t)}{R_{in}}$

$I_f(t) = I_{in}(t) = \frac{V_{in}(t)}{R_{in}}$

$$\boxed{V_{out}(t) = -\frac{1}{R_{in}C_{pix}}\int_0^t V_{in}(t)\,dt}$$

- current voltage for capacitors

$I_f(t) = C_{pixel}\frac{dV_c(t)}{dt}$

$V_c(t) = V_c(t_0) + \int_{t_0}^{t} \frac{I_f}{C_{pixel}}\,dt$

$V_c(t) = V_c(t_0) + \int_{t_0}^{t} \frac{V_{in}(t)}{R_{in}C_{pixel}}\,dt$