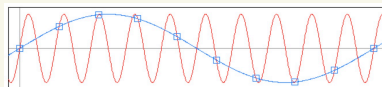


Rasterization = sample points to see if inside triangle

Aliasing = high freq signals are under-sampled

↳ can resemble continuous signal of lower freq



Anti-aliasing = removing high freq signals before sampling

Box blur = ① select kernel size $k \times k$

② for each pixel in img, replace its value w/ AVERAGE of all pixel values in kernel region

③ slide kernel across entire img

↳ simple but FAST

Supersampling = artificially increase sampling rate above sampling frequency

Winding order = order of vertices of $\Delta \rightarrow$ convention is COUNTER clockwise

↳ if neg cross product \rightarrow CW order \rightarrow swap 2 vertices

Nyquist freq = $f_{\text{nyquist}} = \frac{1}{2} f_{\text{sampling}}$ \rightarrow NO aliasing from freqs in signal that are LESS than Nyquist freq

↳ if device can sample at 100Hz \rightarrow Nyquist = 50Hz

$f_{\text{sampling}} > 2f_{\text{signal}}$ f_{sampling} = lowest freq you can sample w/ before aliasing

↳ if signal = 20Hz \rightarrow sampling ABOVE 40 Hz

Transformations

Homogeneous coordinates = represent point in n -dim space w/ $n+1$ coordinates

$$(x, y)^T \rightarrow (x, y, w)^T$$

$$(x, y, z)^T \rightarrow (x, y, z, w)^T$$

$w=1 \rightarrow$ point

$w=0 \rightarrow$ vector

} allows us to represent points & vectors in same coordinate system

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} u & v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} xu + yv + o \\ 1 \end{bmatrix}$$

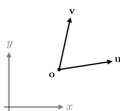
Coordinate System Change

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear



Isometric = preserve distances between every pair of points on object

✓ Rotations, translations, reflections

✗ Scaling, shearing

✗ transformation matrices multiplied RIGHT to LEFT

Flip across y-axis

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Flip across x-axis

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Coordinate System Transformation : $(0, u, v)$ to (x, y)

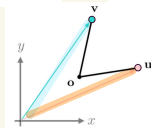
origin
↓
 $(0, u, v)$
↑
2 unit vectors

Point (1,0)

$$\begin{bmatrix} u & v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} u+0 \\ 1 \end{bmatrix}$$

Point (0,1)

$$\begin{bmatrix} u & v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} v+0 \\ 1 \end{bmatrix}$$



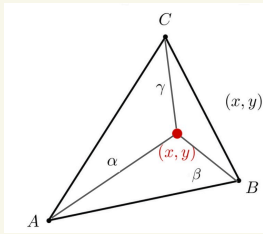
camera transformation: camera (e, u, v)

eye point \uparrow view

transform matrix

$$\begin{pmatrix} r_x & u_x & -v_x & e_x \\ r_y & u_y & -v_y & e_y \\ r_z & u_z & -v_z & e_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

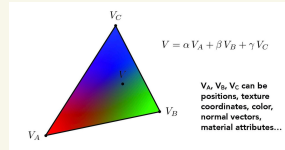
Barycentric coordinates = weighted distance from given point to vertices



$$(x, y) = \alpha A + \beta B + \gamma C$$

$$\alpha + \beta + \gamma = 1$$

used for interpolation



Texture mapping = moving from screen space to texture space (u, v)

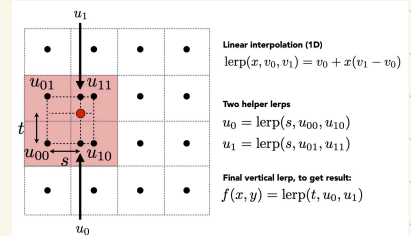
can interpolate texture coordinates (u, v) w/ $u = \alpha u_0 + \beta u_1 + \gamma u_2$

$v = \alpha v_0 + \beta v_1 + \gamma v_2$

Nearest sampling = taking color of texel that's closest to barycentric coordinate

Bilinear sampling = weighted avg of 4 nearest texels w/ 3 LERPs

$$\rightarrow \text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$



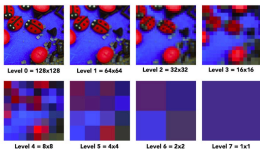
Mip-mapping:

- ① pre-compute lower res versions of texture
- ② store textures in mipmap
- ③ Adaptively choose mipmap level D according to scene

$$D = \log_2 L$$

$$L = \max \left(\sqrt{\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2}, \sqrt{\left(\frac{du}{dy}\right)^2 + \left(\frac{dv}{dy}\right)^2} \right)$$

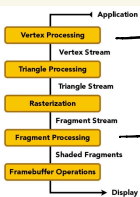
Halve the dimensions each time \rightarrow



- * BIG jump in texture space = far away \rightarrow high level (low res)
- * Small jump in texture space = close up \rightarrow low level (high res)

Bilinear = only use ONE mipmap level

trilinear = LERP between TWO mipmap levels



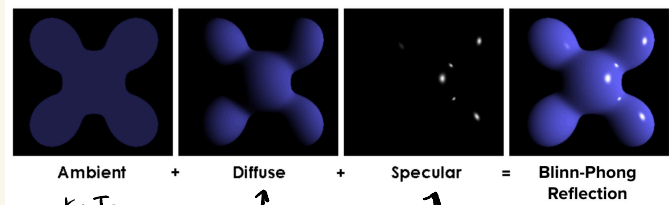
operations on geometry = transformations to screen space

operations on pixels = hidden surface removal / per-fragment shading

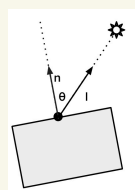
Reflection Model

Blinn-Phong =

↑
default w/
Gouraud
shading



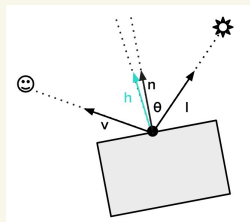
DIFFUSE SHADING
 $K_d \left(\frac{I}{r^2} \right) \max(0, n \cdot l)$



SPECULAR SHADING

$$h = \text{bisector}(v, l) = \frac{v + l}{\|v + l\|}$$

$K_s \left(\frac{I}{r^2} \right) \max(0, n \cdot h)^p$ shininess



Phong shading = interpolate vertex normals per pixel

* SLOW

Gouraud shading = compute light per vertex

* FAST

Hidden surface removal : objects overlap → only display what's visible @ front

* track depth = z-value of fragments

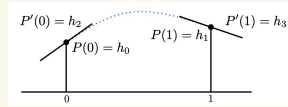
↳ pixel takes value of closest fragment in z-buffer
initialized to infinity

Cubic Hermite Interpolation: combine discrete pts into shape

INPUT: values (p) & derivatives (p') @ endpoints

Output: cubic polynomial that interpolates

Soln: weighted sum of Hermite basis functions



$$P(t) = h_0 H_0(t) + h_1 H_1(t) + h_2 H_2(t) + h_3 H_3(t)$$

$$P(t) = [H_0(t) \ H_1(t) \ H_2(t) \ H_3(t)] \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

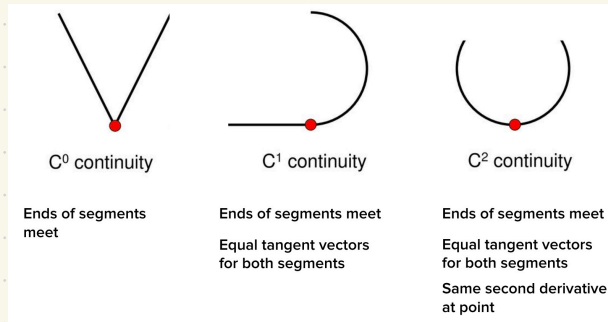
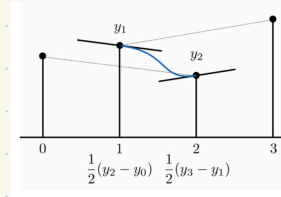
Catmull Rom Interpolation

Input: sequence of points

① calculate slopes between alternating pts

② use Hermite interpolation

Output: spline w/ C^1 continuity



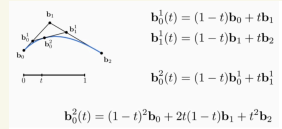
~~review~~ review math

Bezier Curves

Cubic Bezier: specify derivatives w/ control points

de Casteljau Algorithm = recursively compute intermediate control points through loop

$$P_i' = (1-t)P_i + t(P_{i+1})$$



Catmull Rom = easy to use when you know set of ^{control} points ahead of time → ensures curve passes through ALL control pts (except maybe endpoints)

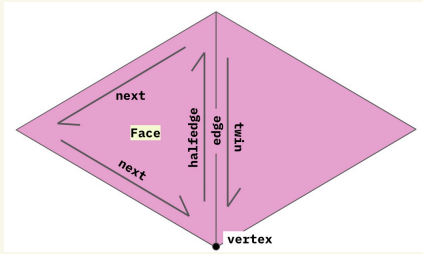
Halfedges = represent meshes to represent 3D shapes

```
struct HalfEdge {
    HalfEdge *twin;
    HalfEdge *next;
    Vertex *vertex;
    Edge *edge;
    Face *face;
}

struct Vertex {
    Point pt;
    HalfEdge *halfedge;
}

struct Edge {
    HalfEdge *halfedge;
}

struct Face {
    HalfEdge *halfedge;
}
```



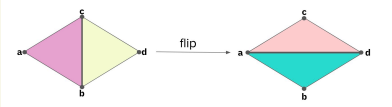
* useful to use do-while loops
 * $\rightarrow \text{next}()$ to get next edge
 $\rightarrow \text{twin}()$ to get opposite halfedge

* REVIEW coding examples!!! *

Edge flip = flip a half edge

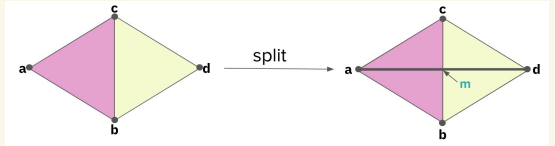
* no elements created or destroyed *

Triangles $(a, b, c), (b, d, c)$ become $(a, d, c), (a, b, d)$:

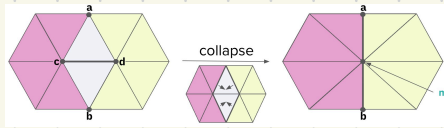


Edge split = insert midpoint into halfedge

* elements are added *



Edge collapse = replace edge (c, d) w/ vertex m
 * delete elements * (lose 2 faces for Δ)

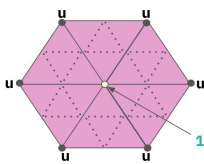


subdivision : coarse mesh \rightarrow smooth algorithmically

LOOP subdivision = for triangle meshes

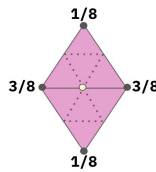
- ① split each Δ face into 4 \rightarrow new Δ , new vertices
- ② update old & new vertex positions as weighted sum

n : vertex degree
 u : $3/16$ if $n = 3$,
 $3/(8n)$ otherwise



Old vertices

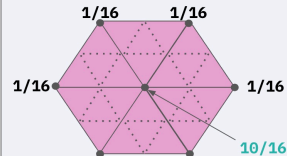
$$v'_{old} = (1 - nu)v_{old} + \sum_{v_j \in N(v_{old})} uv_j$$



New vertices

$$v'_{new} = \frac{3}{8}(v_{left} + v_{right}) + \frac{1}{8}(v_{up} + v_{down})$$

Example: degree 6



- ③ for any new edge that connects new to old vertex \rightarrow EDGE FLIP

Catmull CLARK SUBDIVISION: meshes w/ variable polygon

- ① Add vertex in each face
- ② Add midpoint to each edge
- ③ Connect all new vertices
- ④ Adjust vertex positions to weighted avg

RAY TRACING = traces path of light as rays that travel through scene

- ① Cast ray from camera into scene
- ② Check whether ray intersects any objects in scene
- ③ Determine how intersecting surface should be shaded
- ④ If object reflective → generate NEW ray reflecting off surface & trace that ray
↳ If object transparent: refraction occurs & Snell's Law to bend light &
- ⑤ shadow rays: rays cast from pt of intersection toward each light source
- ⑥ Global illumination: where light bounces off surfaces → indirect lighting

position of origin
direction of ray
time

$$r(t) = o + t d$$

Plane equation: $(p - p') \cdot N = 0$

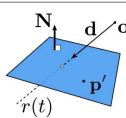
any pt on plane normal vector

RAY-PLANE INTERSECTION:

Ray equation:
 $r(t) = o + t d, 0 \leq t < \infty$

Plane equation:
 $p : (p - p') \cdot N = 0$

Solve for intersection
Set $p = r(t)$ and solve for t
 $(p - p') \cdot N = (o + t d - p') \cdot N = 0$
 $t = \frac{(p' - o) \cdot N}{d \cdot N}$ Check: $0 \leq t < \infty$



$$(p - p') \cdot N = 0$$

$$(o + t d - p') \cdot N = 0$$

$$(o - p') \cdot N + t d \cdot N = 0$$

$$t = \frac{(p' - o) \cdot N}{d \cdot N}$$

$t < 0 \rightarrow$ intersection BEHIND origin so
INVALID intersection

$d \cdot N = 0 \rightarrow$ direction perpendicular to plane's
normal vector = ray is PARALLEL

$(o - p') \cdot N = 0 \rightarrow$ infinite intersections
 $(o - p') \cdot N \neq 0 \rightarrow$ zero intersections

RAY SURFACE INTERSECTION given $f(x, y, z)$ and $r(t)$

- ① Set $f(o + t d) = 0$ and solve for t
- ② Plug values of t back into ray equation
- ③ ID where ray first hits surface

$$f(x, y, z) = \frac{(x-2)^2}{4} + (y-2)^2 + \frac{z^2}{4} - 1$$

$$r(t) = (0, 0, 0) + t(1, 1, 0)$$

sub back into here

$$\begin{aligned} x &= 0 + t \cdot 1 = t \\ y &= 0 + t \cdot 1 = t \\ z &= 0 + t \cdot 0 = 0 \end{aligned}$$

$$\begin{aligned} \frac{(x-2)^2}{4} + (y-2)^2 + \frac{z^2}{4} - 1 &= 0 \\ \frac{(t-2)^2}{4} + (t-2)^2 + \frac{0^2}{4} - 1 &= 0 \\ \frac{5}{4}(t-2)^2 - 1 &= 0 \\ (t-2)^2 &= \frac{4}{5} \end{aligned}$$

$$t = 2 \pm \sqrt{\frac{4}{5}}$$

plug first t back into ray equation

$$\begin{aligned} r(t) &= (0, 0, 0) + (2 - \sqrt{\frac{4}{5}})(1, 1, 0) \\ &= (2 - \sqrt{\frac{4}{5}}, 2 - \sqrt{\frac{4}{5}}, 0) \end{aligned}$$

first intersection point

RAY-TRIANGLE INTERSECTION

$$\begin{aligned}
 P &= \alpha P_0 + \beta P_1 + \gamma P_2 \longrightarrow P = (1-b_1-b_2)P_0 + b_1P_1 + b_2P_2 \\
 \text{Let } b_1 &= \beta \\
 b_2 &= \gamma \longrightarrow \alpha = 1-b_1-b_2 \\
 \text{point within triangle} & \\
 \downarrow \text{Plug in } P &= O + tD \\
 O + tD &= P_0 + b_1(P_1 - P_0) + b_2(P_2 - P_0) \\
 O - P_0 &= tD + b_1(P_1 - P_0) + b_2(P_2 - P_0) \\
 \begin{bmatrix} -D & P_1 - P_0 & P_2 - P_0 \end{bmatrix} \begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} &= O - P_0
 \end{aligned}$$

Möller-Trombore Algorithm = efficient way to determine if ray intersects \triangle

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{s_1 \cdot e_1} \begin{bmatrix} s_2 \cdot e_2 \\ s_1 \cdot s \\ s_2 \cdot D \end{bmatrix}$$

$$\begin{aligned}
 t &\geq 0 \\
 0 &\leq b_1 \leq 1 \\
 0 &\leq b_2 \leq 1 \\
 0 &\leq 1 - b_1 - b_2 \leq 1
 \end{aligned}$$

Acceleration

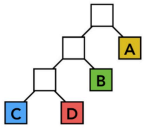
Bounding Volume Hierarchy = organized objects into tree structure where each node has bounding volume which encapsulates set of objects / primitive

Goal: reduce # of objects that need to be checked for intersections

if ray does NOT intersect bounding volume \rightarrow skip checking individual objects within it

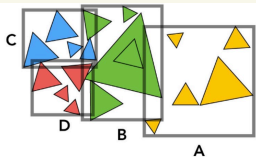
Internal nodes:

1. Bounding box.
2. Reference to children.



Leaf nodes:

1. Bounding box.
2. List of primitives in the box.

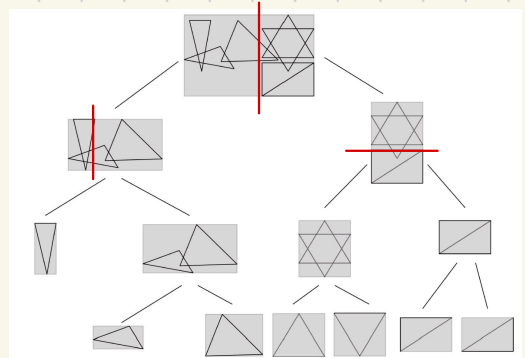


Intersect (Ray ray, BVH node)

```

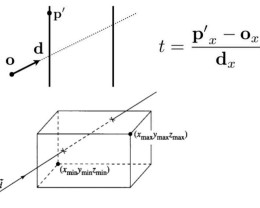
if (ray misses node.bbox) return;
if (node is a leaf node)
    test intersection with all objs;
    return closest intersection;
hit1 = Intersect (ray, node.child1);
hit2 = Intersect (ray, node.child2);
return closer of hit1, hit2;
    
```

- ① Always pick the longest axis to divide
- ② Use center of mass to decide their relative pos
- ③ Keep BVH balanced \rightarrow try to ensure same # of triangles for children nodes






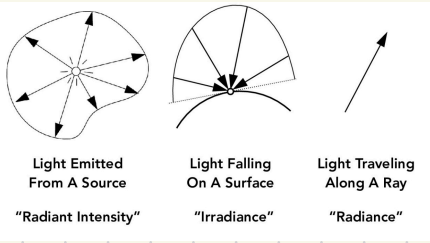
Hint: Axis-aligned ray-plane intersection equation

Perpendicular to x-axis



Radiometry

Flux (power)	watts (W)	$\Phi = \frac{dQ}{dt}$	total energy per time
Radiant Intensity	flux / solid angle (W / sr)	$I(\omega) = \frac{d\Phi}{d\omega}$ 	amt of radiant flux in specific direction per solid angle (think flashlight)
Irradiance	flux / area / solid angle (W/m ²)	$E(x) = \frac{d\Phi(x)}{dA} \quad E = \frac{\Phi}{A} \cos\theta$ 	how much power is received by a surface (sunlight hitting solar panel)
Radiance	flux / area (W / sr m ²)	$L(p, \omega) = \frac{d^2\Phi(p, \omega)}{d\omega dA \cos\theta}$ 	how much light is traveling from in a specific direction from a surface (computer screen from diff angles)



Symbol/Name	Radiometry Unit/Name	Photometry Unit/Name	Effect of Increased R
Q: Energy	Radiant Energy Joules (W-s)	Luminous Energy Lumen-sec	=
Φ : Flux (Power)	Radiant Power W	Luminous Power Candela-sr	=
I: Angular Flux Density	Radiant Intensity W/sr	Luminous Intensity Candela = Lumen/sr	=
E: Spatial Flux Density	Irradiance (in), Radiosity (out) W/m ²	Illuminance (in), Luminosity (out) Lux = Lumen/m ²	↓
L: Spatio-Angular Flux Density	Radiance W/m ² /sr	Luminance Nit = Candela/m ²	=

Physics Symbol/Name	Radiometry Unit/Name	Photometry Unit/Name	Definition
Q Energy	Radiant Energy Joules (W-s)	Luminous Energy Lumen-sec	$Q = \int_{t_0}^{t_1} \Phi dt$
Φ Flux (Power)	Radiant Power W	Luminous Power Lumen (Candela-sr)	$\Phi = \frac{dQ}{dt}$
I Angular Flux Density	Radiant Intensity W/sr	Luminous Intensity Candela (Lumen/sr)	$I(\omega) = \frac{d\Phi}{d\omega}$
E Spatial Flux Density	Irradiance (in), Radiosity (out) W/m ²	Illuminance (in), Luminosity (out) Lux (Lumen/m ²)	$E(p) = \frac{d\Phi(p)}{dA}$
L Spatio-Angular Flux Density	Radiance W/m ² /sr	Luminance Nit (Candela/m ²)	$L(p, \omega) = \frac{d^2\Phi(p, \omega)}{d\omega dA \cos\theta}$ $= \frac{dE(p)}{d\omega \cos\theta} = \frac{dI(p, \omega)}{dA \cos\theta}$

Lambert's Law

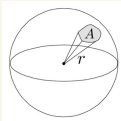


power / unit area proportional to $\cos\theta = \mathbf{l} \cdot \mathbf{n} \rightarrow E = \frac{\Phi}{A} \cos\theta$

Solid angle = measure of how large object appears from given point in 3D space

↳ ratio of subtended area on sphere to radius squared

$$\Omega = \frac{A}{r^2}$$

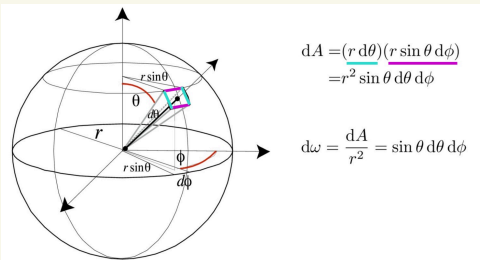


sphere = 4π steradians
hemisphere = 2π steradians

Regular angle = ratio of subtended arc length on circle to radius

$$\theta = \frac{l}{r}$$

circle = 2π radians



Probability

$$\text{PMF} = P[X=x]$$

$$\text{Expectation: } E[X] = \sum_i x_i p_i = \int x p(x) dx$$

$$\text{CDF} = F(x) = \int_{-\infty}^x p(t) dt$$

Inversion Method: ① Calculate CDF: $F(x) = \int_{-\infty}^x p(t) dt$

② Invert CDF: $F^{-1}(x) \xrightarrow{\text{Set } U = \text{CDF and solve for } x}$ becomes inverse

③ sampling X according to $p(x)$
achieved by sampling $U \rightarrow X = F^{-1}(U)$

Monte Carlo Integration

$$F_N = \frac{1}{N} \sum_{i=1}^N \frac{f(x_i)}{p(x_i)} \rightarrow E[F_N] = \int_a^b f(x) dx$$

* good to approximate complex shapes
* don't need a lot of samples

for importance sampling \rightarrow need to map to $[0,1]$ ($u \rightarrow x$)

$$x_i = -\frac{\pi}{2} \rightarrow u_i = 0$$

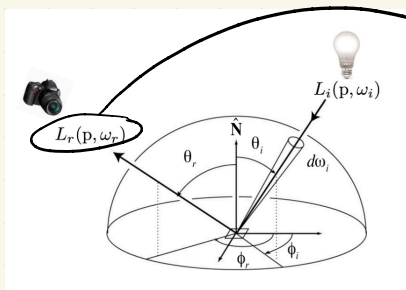
$$x_i = \frac{\pi}{2} \rightarrow u_i = 1$$

$$u_i = \frac{x_i}{\pi} + \frac{1}{2}$$

Light Transport

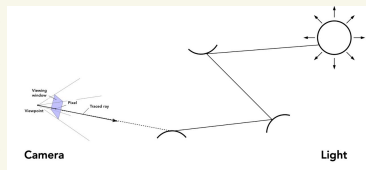
BRDF = bidirectional reflectance distribution function

* ratio of reflected radiance in given outgoing direction ω_o to incoming irradiance from dir ω_i



$$L_r(p, \omega_o) = L_e(p, \omega_o) + \int_{\Omega^+} f_r(p, \omega_i, \omega_o) L_i(p, \omega_i) \cos \theta_i d\omega_i$$

Global Illumination = recursively calculate how much light falls onto point p



1 bounce: radiance from light source $\rightarrow p \rightarrow$ camera

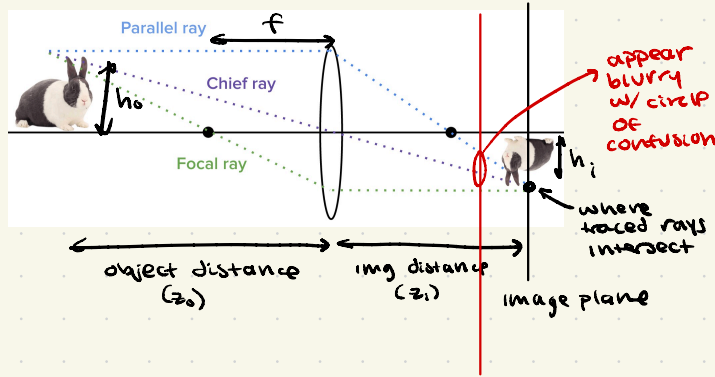
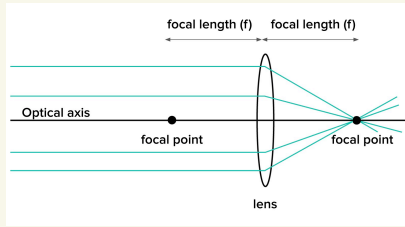
2 bounce: radiance from light source $\rightarrow p' \rightarrow p \rightarrow$ camera

Termination Conditions

* Russian Roulette: at each bounce, randomly terminate current ray w/ $p(x) = 1 - p_{rr}$

Importance sampling = sampling MORE from important areas \rightarrow reduces variance

Cameras & lenses

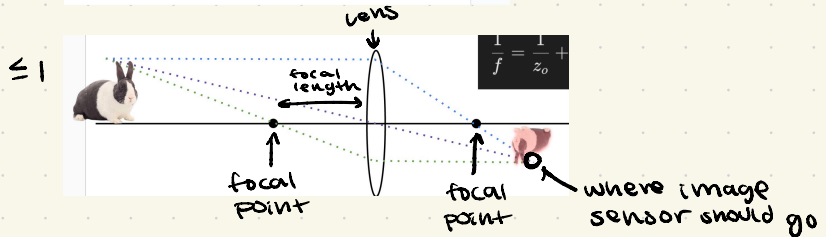
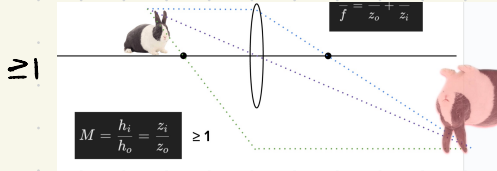


Thin Lens Equation:

$$\frac{1}{f} = \frac{1}{z_o} + \frac{1}{z_i}$$

Magnification

$$M = \frac{h_i}{h_o} = \frac{z_i}{z_o}$$



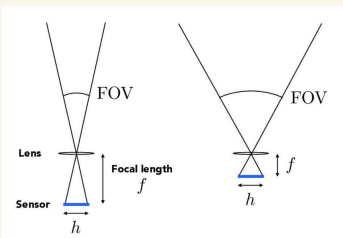
chief ray = pass through CENTER of lens

focal ray = emerges parallel to optical axis

parallel ray = directed through focal point on other side

focal point = where img is formed by the lens

FOV = angle of scene captured by camera lens



smaller focal length → LARGER FOV

larger focal length → SMALLER FOV

smaller sensor size → smaller FOV

$$\text{FOV} = 2 \arctan\left(\frac{h}{2f}\right)$$

h = sensor height
 f = focal length

* can use similar triangles w/ $\frac{h}{f}$

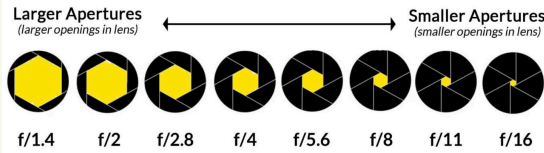
circle of confusion = optical spot caused by cone of light not coming to focus

smaller aperture \rightarrow more concentrated rays \rightarrow more depth of field (less blur)
= BIGGER f stop

large aperture \rightarrow shallow depth of field

smaller f-stop \rightarrow shallower depth of field

Exposure = irradiance \times time \times gain



Larger aperture = higher irradiance

$$F\# = \frac{\text{focal length}}{\text{diameter of aperture}}$$

$\sqrt{2}$ increase in aperture DOUBLES light

$\times 2$ shutter duration $\rightarrow \times 2$ exposure

$\times 2$ ISO gain $\rightarrow \times 2$ exposure

shutter duration = seconds sensor exposed to light

ISO = sensor's sensitivity to light = how much light amplified

higher ISO \rightarrow more noise

Simulation

Euler's Method: $\frac{dx}{dt} = f(x, t)$

x^t = position

\dot{x}^t = velocity

\ddot{x}^t = acceleration

$$x^{t+\Delta t} = x^t + \Delta t \dot{x}^t$$

$$\dot{x}^{t+\Delta t} = \dot{x}^t + \Delta t \ddot{x}^t$$

✓ simple & easy to compute

✗ errors accumulate

↳ small step size to have low approximation error

Implicit Euler's Method / Backward Euler's

$$x^{t+\Delta t} = x^t + \Delta t \dot{x}^{t+\Delta t}$$

$$\dot{x}^{t+\Delta t} = \dot{x}^t + \Delta t \ddot{x}^{t+\Delta t}$$

more stable

non-linear equations = DIFFICULT

Modified Euler's

$$x^{t+\Delta t} = x^t + \frac{\Delta t}{2}(\dot{x}^t + \dot{x}^{t+\Delta t})$$

$$\dot{x}^{t+\Delta t} = \dot{x}^t + \Delta t \ddot{x}^t$$

more stable

Runge-Kutta Integration

$$x^{t+\Delta t} = x^t + \Delta t \dot{x}^t + \frac{1}{2}(\Delta t)^2 \ddot{x}^t$$

$$\dot{x}^{t+\Delta t} = \frac{x^{t+\Delta t} - x^t}{\Delta t}$$

dissipates energy

Hooke's Law:

$$f_{a \rightarrow b} = k_s \frac{b-a}{\|b-a\|} (\|b-a\| - l)$$

\uparrow spring constant \uparrow length

$$f_{b \rightarrow a} = f_{-a \rightarrow b}$$

$$F = ma$$

Animation

Forward kinematics = angles for joints \rightarrow computer determines final position

Inverse kinematics = ending position \rightarrow compute joint angles to reach position

X no realistic soln

X multiple possible solns \rightarrow unique depending on how θ constrained

Keyframes = important moments in some transition / motion in between start / end

* usually interpolate between them

linear interpolation \neq rotations

\uparrow
straight line

\uparrow
circle

Not good match for

light

Spectral power distribution = non-negative function giving power in light beam @ given wavelength

* characterizes light source

* ADDITIVE!

* watts / nm

Monospectral Distribution = single color

$$\boxed{R \cdot s_R(\lambda) + G \cdot s_G(\lambda) + B \cdot s_B(\lambda)} = \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = S_{\text{disp}}(\lambda)$$

\uparrow red brightness [0,1] \uparrow red monospectrum

Cone cells = diff sensitivities

$$\begin{aligned}
 L &= \text{orange / yellow} = \int r_L(\lambda) s(\lambda) d\lambda \\
 M &= \text{green / yellow} = \int r_M(\lambda) s(\lambda) d\lambda \\
 S &= \text{blue} = \int r_S(\lambda) s(\lambda) d\lambda
 \end{aligned}$$

\uparrow response curve \uparrow SPD

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} -r_S- \\ -r_M- \\ -r_L- \end{bmatrix} \begin{bmatrix} 1 \\ s \\ 1 \end{bmatrix}$$

metamer = 2 diff spectra w/ same visual (S,M,L) response
 * reproduce real-world scenes w/ HARD to recreate spectra

$$\begin{array}{c} \text{color we perceive on display} \end{array} \begin{bmatrix} - & r_S & - \\ - & r_M & - \\ - & r_L & - \end{bmatrix} \begin{bmatrix} | & & | \\ s_R & s_G & s_B \\ | & & | \end{bmatrix} \begin{array}{c} \text{color we perceive in real life} \end{array} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{array}{c} \text{color we perceive on display} \end{array} \begin{bmatrix} - & r_S & - \\ - & r_M & - \\ - & r_L & - \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

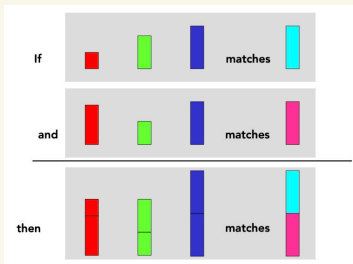
Gamut = range of colors that can be displayed on a device

*** device dependent ***

* color matching w/ primary lights

↳ not possible = add NEG amt of color into test side

* color matching = LINEAR



3 primary colors = necessary for normal color vision

2 primary colors = red green colorblindness

Human Eye

Rods = primarily in low light, diff shades of gray

cones = "photopic" receptors → sensation of color

Hue, Saturation, Value (HSV)

Hue = dominant wavelength (what color)

Saturation = how vivid the color is

value = amount of light

chromaticity diagram

* pure saturated spectral
 at corners, desaturated in center
 * does NOT include black

CIE LAB = strives for perceptual uniformity

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\ r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\ r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B \end{bmatrix}^{-1} \begin{bmatrix} - & r_S & - \\ - & r_M & - \\ - & r_L & - \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

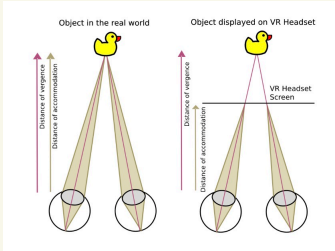
negative color values = gamut not enough to display soln

Virtual Reality

vergence = rotation of eyes to focus on near/far objects

accommodation = eye's ability to change shape of lens to bring objects into focus
 * lens accommodate physical screen distance, NOT virtual depth

vergence - accommodation conflict = vergence cues of virtual distance
 accommodation cues fixed at display distance
MISMATCH SIGNALS = visual discomfort

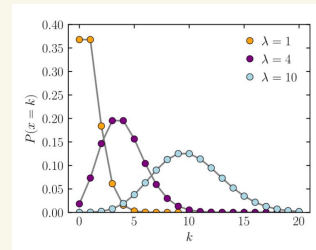


- * low-latency tracking/rendering
- * monoscopic = both eyes w/ same 360° frame → no stereoscopic depth
- * MORE FOV = enhanced realism → can reduce angular resolution
- * real+virtual objects = mixed reality

Sensors

- * photons enter according to Poisson distribution
 - ↳ occurrence of indep events over fixed time interval
 - ↳ avg rate of arrival = λ

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$$



low light = fewer photons arrive

signal has more variability → Poisson shot noise

Aperture										
	F32	F22	F16	F11	F8	F5,6	F4	F2,8	F2	F1,4
Shutter										
	1/1000	1/500	1/250	1/125	1/60	1/30	1/15	1/8	1/4	1/2
Gain										
	ISO 50	ISO 100	ISO 200	ISO 400	ISO 800	ISO 1600	ISO 3200	ISO 6400	ISO 12800	ISO 25600